Influence of statistical fluctuations on K/π ratios in relativistic heavy ion collisions

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The influence of pure statistical fluctuations on K/π ratio is investigated in an event-by-event way. Poisson and the modified negative binomial distributions are used as the multiplicity distributions since they both have statistical background. It is shown that the distributions of the ratio in these cases are Gaussian, and the mean and relative variance are given analytically.

Key words: K/π ratio, statistical fluctuations, relativistic heavy ion collision

The ultimate goal of current and future ultra-relativistic heavy ion program is the production and characterization of an extended volume of deconfined phase of quarks and gluons, the quark-gluon plasma (QGP). The possible existence of QGP as the equilibrium state of strongly interacting matter has been predicted in lattice theories [1] at sufficiently high temperature and density. So QGP might exist in the early universe and inside the neutron stars. Heavy ion collision at extremely high energies is the only chance to study QGP in a controlled way. In laboratory experiments, however, the QGP can survive only for very short period of time, and it will cool during its expanding and become hadronic matter at freeze-out point. It is very difficult to detect QGP in experiments because most events are, due to collision geometry and quantum fluctuations, without QGP even if the condition for QGP creation can be reached in the experiments. So, a variety of possible signatures for the transient existence of the deconfined state of matter in the collisions have been proposed theoretically and experimentally (see [2, 3, 4] for a review). One of the most important and most interesting signatures is the enhanced strangeness production in the process of heavy ion collisions [5] compared with that in $p-p$ collisions. The investigation of strangeness production is useful for understanding the mechanism of heavy quark production since no strangeness content exists in the initial state in heavy-ion collisions. Strangeness production can also shed light on the time scale of chemical freeze-out and therefore can carry information about the early stage of a heavy ion collision [6].

Great progress has been made with the proposal and applications of event-by-event analysis [7, 8, 9, 10, 11, 12, 13] in the study of high energy heavy ion collisions. Event-by-event analysis of heavy ion collisions became possible with the advent of large acceptance detectors [14]. The philosophy of event-by-event physics is based on following speculation: Although the conditions to produce QGP may be reached in every event, the fact that a phase transition is a critical phenomenon implies that it may occur only in a very small sub-sample of events. So the fluctuations accompanying the phase transition will, in effect, be averaged out in the conventional ensemble analyses. The event-by-event analysis searches for fluctuations of observables at the event level, so it can be used to select interesting or anomalous event candidates [15] with specific dynamical properties. So the new method can provide dynamical information which cannot be obtained from the traditional inclusive spectra.

Recently the event-by-event analysis is used in experiments to analyze strangeness productions in high energy heavy ion collisions. In [16, 17] the kaon to pion ratio for single event is studied as the event observable with which one can look at fluctuations in the chemical freeze-out stage of the collisions. Of course, the distribution of the K/π ratio is the result of a combination of statistical and possible dynamical fluctuations in the strangeness production. The method of mixed events, in which there are no momentum correlation among particles by construction, is used in the experimental analyses to estimate the effect of the former on the distribution of K/π ratio. Both distributions for K/π ratio from the real data and the mixed events are Gaussian, and the widths are only with a little difference. That means the dominance of statistical fluctuations in strangeness production. Although it is well established that there is no spatial-temporal correlation in the mixed events, we do not know exactly if there exist some other correlations. In the study of K/π ratio the only quantities concerned are the numbers of produced kaons and pions. The event-mixing technique will not change the total numbers of the particles produced. These numbers carry some important information about the chemical fluctuations at the freeze-out point. So, some information about the strangeness production may still be retained in the mixed events. To investigate the problem theoretically further one should ask: What is the distribution of K/π ratio if there exist only statistical multiplicity fluctuations in the collisions? Will it be a Gaussian as showed from the mixed events? If yes, what are the mean and the width? Are they in agreement with those obtained from the mixed events? What will be the dependences of them on mean multiplicities of kaons and pions?

In this paper the K/π ratio for the case with pure statistical fluctuations is studied in an event-by-event way with two assumptions: (1) There is no dynamical fluctuation in the productions of both the kaons and pions, and (2) there is no correlation between the productions of the two kinds of particles. This is, of course, a trivial case, but it is the base for any event-by-event investigation on the influence of dynamical fluctuations on K/π ratio.

Since we have assumed no dynamical fluctuations in the productions of kaons and pions nor correlation between

them, the distributions of the multiplicities of kaons and pions can be specified from some statistical considerations. Two possibilities will be considered in this paper. As a first possibility, the distributions can be Poissonian, i.e. the probabilities with which there are k kaons and/or π pions in an event are, respectively,

$$p_K = \frac{\langle k \rangle^k}{k!} e^{-\langle k \rangle}, \qquad p_\Pi = \frac{\langle \pi \rangle^\pi}{\pi!} e^{-\langle \pi \rangle} \tag{1}$$

with $\langle k \rangle$ and $\langle \pi \rangle$ the mean multiplicities of kaons and pions in the collisions. In a global analysis, the global K/π ratio is then

$$R_g = \langle k \rangle / \langle \pi \rangle \quad . \tag{2}$$

From the view point of the theory of coherent state the assumption of Poisson distributions for the multiplicities implies that both kaons and pions are emitted from some coherent sources and that no thermal effect is taken into account. Now we turn to event-by-event analysis of the ratio. For an event with k kaons and π pions the K/π ratio is

$$r_e \equiv \frac{k}{\pi} \, , \tag{3}$$

and the probability density for K/π ratio to be in neighborhood of r_e can be given by

$$P_{r_e} = \sum_{r_e = k/\pi} p_K(k) p_\Pi(\pi) = \sum_{\pi=1}^{\infty} \frac{\langle k \rangle^{r_e \pi}}{(r_e \pi)!} \frac{\langle \pi \rangle^\pi}{\pi!} e^{-(1+R_g)\langle \pi \rangle} \quad , \tag{4}$$

because the productions of kaons and pions have been assumed to be independent.

Since p_K and p_Π are peaked at $\langle k \rangle$ and $\langle \pi \rangle$ respectively, it is easy to see that P_{r_e} will be peaked near R_g. A natural question is: Can P_{r_e} be Gaussian for given $\langle k \rangle$ and $\langle \pi \rangle$? Since Eq. (4) is too complicated, Monte Carlo simulation can be used to answer this question. To get an event for this case, one needs only to generate two independent random integer numbers (according to two Poisson distributions with means $\langle k \rangle$ and $\langle \pi \rangle$ respectively). The random integer number corresponding to $\langle k \rangle$ is taken to be the kaon multiplicity in the event, the other for pions. Then one gets a corresponding K/π ratio r_e for the event. The distribution of r_e from the simulation is given in Fig. 1 with 1 million events used. In the simulation the mean multiplicities for kaons and pions are chosen to be 38 and 200 respectively for illustration. Here we choose $R_g = 0.19$, the same as shown experimentally in Pb-Pb collisions at 158 A GeV, for later comparison with experimental result. One can see that the distribution of the K/π ratio is indeed approximately a Gaussian. This is consistent with the conclusions given from the mixed events in Refs. [16, 17]. To get more information about the distribution one may want to know the mean and width of the distribution. From Eq. (4) one can see that the distribution of r_e is not symmetric about R_g, so the mean value of the distribution is, to some extent, different from R_g. The mean value of the distribution for the trivial case can be calculated analytically as

$$R_e = \langle \frac{k}{\pi} \rangle \equiv \sum_{k=0}^{\infty} \sum_{\pi=1}^{\infty} \frac{k}{\pi} p_K(k) p_\Pi(\pi) \left/ \sum_{k=0}^{\infty} \sum_{\pi=1}^{\infty} p_K(k) p_\Pi(\pi) \right. \tag{5}$$

$$\simeq R_g \sum_{\pi=1}^{\infty} \frac{\langle \pi \rangle^{\pi+1}}{\pi \pi!} e^{-\langle \pi \rangle} \, . \tag{6}$$

In the transition from Eq. (5) to Eq. (6) condition $\langle \pi \rangle \gg 1$ has been used. The relative accuracy in Eq. (6) is of the order of $e^{-\langle \pi \rangle}$, thus the approximation is quite good for $\langle \pi \rangle$ a few hundreds which is quite normal in current high energy heavy ion collisions. By using the generating function for the Poisson distribution

$$G_\Pi(x) = \sum_{\pi=0}^{\infty} \frac{\langle \pi \rangle^\pi x^\pi}{\pi!} e^{-\langle \pi \rangle} = \sum_{\pi=1}^{\infty} x^\pi p_\Pi(\pi) = e^{\langle \pi \rangle (x-1)} \quad , \tag{7}$$

the mean value of the distribution R_e can be expressed as

$$R_e = R_g \int_0^1 dx \langle \pi \rangle \frac{G_\Pi(x) - G_\Pi(0)}{x} \quad . \tag{8}$$

We are particularly interested in the ratio R between R_e and R_g, which is a measure of the deviation of the two K/π ratios obtained from global and event-by-event analyses, respectively. In our trivial case

$$R \equiv \frac{R_e}{R_g} = \langle \pi \rangle \int_0^1 dx \frac{G_\Pi(x) - G_\Pi(0)}{x} \quad , \tag{9}$$

which depends only on the mean multiplicity of pions. It can be seen that R will approach 1.0 when $\langle \pi \rangle \to \infty$. In this limit the mean values of K/π ratios from global and event-by-event analyses are equal, as can be naively expected. The change behavior of R as a function of $\langle \pi \rangle$ is shown in Fig. 2. R is large for lower mean pion multiplicity and decreases with $\langle \pi \rangle$. But the values of R are always close to the trivial value 1.0 within an accuracy of about 1% in the graphed range of the mean pion multiplicity. So one can hardly find the difference between R_e and R_g. Using the generating function for pion multiplicity distribution, one can also calculate the width of the K/π ratio distribution. First one can get

$$\left\langle \left(\frac{k}{\pi} \right)^2 \right\rangle = \langle k^2 \rangle \int_0^1 dx \ \frac{G_\Pi(x) - G_\Pi(0)}{x} \ln \frac{1}{x} \quad , \tag{10}$$

where $\langle k^2 \rangle = \langle k \rangle^2 + \langle k \rangle$, since the distribution of the kaon multiplicity is assumed to be a Poissonian. The width of the K/π ratio distribution is then

$$\sigma = \sqrt{\left\langle \left(\frac{k}{\pi} \right)^2 \right\rangle - \left\langle \frac{k}{\pi} \right\rangle^2} \quad . \tag{11}$$

The relative variance is then

$$\frac{\sigma}{R_e} = \sqrt{\frac{\left(1 + \frac{1}{R_g \langle \pi \rangle} \right) \int_0^1 dx \ \ln \frac{1}{x} \ \frac{G_\Pi(x) - G_\Pi(0)}{x}}{\left[\int_0^1 dx \ \frac{G_\Pi(x) - G_\Pi(0)}{x} \right]^2} - 1} \quad . \tag{12}$$

Different from R the relative variance depends on both $\langle \pi \rangle$ and R_g (thus on $\langle k \rangle$). The behavior of the relative variance as a function of $\langle \pi \rangle$ for given $R_g = 0.19$ is shown in Fig. 3. The value of R_g is chosen for illustration and is the same as that obtained from current experiments on Pb-Pb collision at 158 A GeV. For $\langle \pi \rangle$ from 100 to 300 the relative variance decreases quite quickly from about 28% to about 16.5%. The relative variance given experimentally from the mixed events is about 23% at $\langle \pi \rangle$=270.13 [18]. Since the relative variance depends on R_g, different curve for the dependence of σ/R_e on $\langle \pi \rangle$ will be obtained if another value of R_g is taken as input. For fixed $\langle \pi \rangle$, the larger R_g, the smaller σ/R_e. The decrease tendency of σ/R_e with $\langle \pi \rangle$ will be similar for all R_g. The decrease of the relative width σ/R_e with the increase of $\langle \pi \rangle$ can be anticipated from the fact that both the distributions for kaons and pions are of Poisson in which the relative widths of the multiplicity distributions are inversely proportional to the root of the mean multiplicities. In Refs. [16, 17] the mean multiplicity of positively charged pions is 270.13, in the range discussed in this paper. The experimental point is given in Fig. 3 by a solid square. One can see that the theoretically calculated relative width of the K/π ratio distribution from the Poissonian multiplicity distributions would be too small. The discrepancy of calculated σ/R_e from the experimental result for the same $\langle \pi \rangle$ shows the important role played by the decoherent effect in the emission of the pions and kaons. The decoherent effect will broaden the multiplicity distributions of kaons and pions, so that a larger relative width of the K/π ratio distribution can be expected. Such thermal effect cannot be washed out in the so called mixed events.

As the second possibility for the multiplicity distribution with pure statistical fluctuations, one can work with the modified negative binomial (MNB) distribution [19, 20]. As a generalization of the negative binomial distribution, the thermal effect plays an important role in the MNB distribution. In this case, the generating function is

$$G_{\mathrm{MNB}}(x) = \left(\frac{1 - \Delta(x - 1)}{1 - r(x - 1)} \right)^k \quad , \tag{13}$$

with three parameters Δ, r and k. As shown in the first paper in [19], k can be interpreted as the maximum number of fireballs or clusters in some initial states, Δ and r can be related to the rates of birth and immigration processes. The mean multiplicity in the MNB is $\langle n \rangle = k(r - \Delta)$. The width squared of the multiplicity distribution is $w \langle n \rangle \equiv \langle n^2 \rangle - \langle n \rangle^2 = \langle n \rangle (1 + r + \Delta)$. With the MNB distributions for multiplicities of kaons and pions, the distribution of event-by-event K/π ratio is shown to be also a Gaussian, similar to but broader than that given in

Fig. 1. Our calculations show that the mean value and the width of K/π ratio distribution have weak dependence on the choice of k. Thus, the parameter k is chosen to be 200 in the following. In the calculation of the dependences of the mean value and width of the K/π ratio distribution on the pion mean multiplicity, there are two free parameters w_π and $w_k = (\langle k^2 \rangle - \langle k \rangle^2)/\langle k \rangle$ which represents the relative width of the distribution of kaon multiplicity. Then, R can be calculated from Eq. (9) with G_{MNB} in place of G_Π. The width can also be calculated as

$$\frac{\sigma}{R_e} = \sqrt{\frac{\left(1 + \frac{w_k}{R_g \langle \pi \rangle}\right) \int_0^1 dx \, \frac{G_{\mathrm{MNB}}(x) - G_{\mathrm{MNB}}(0)}{x} \ln \frac{1}{x}}{\left[\int_0^1 dx \, \frac{G_{\mathrm{MNB}}(x) - G_{\mathrm{MNB}}(0)}{x}\right]^2} - 1} \quad , \tag{14}$$

We do not write w_k in Eq. (12) as it is simply 1, because there the kaon multiplicity distribution is assumed to be a Poissonian. Data from NA49 experiments show that $(\langle N^2 \rangle - \langle N \rangle^2)/\langle N \rangle$ is about 2.0 for all charged particles. Considering the fact that most (about 90%) charged particles are pions, one can fix approximately $w_\pi = (\langle \pi^2 \rangle - \langle \pi \rangle^2)/\langle \pi \rangle = 2.0$ for comparison with the experiment. Numerical calculations show that σ/R_e has a quite weak dependence on w_π. From the MNB distribution one can get the two parameters in Eq. (13) for pion multiplicity distribution as $r = (w_\pi - 1 + \langle \pi \rangle/k)/2$, $\Delta = (w_\pi - 1 - \langle \pi \rangle/k)/2$ with k preset to be 200. Dependence of σ/R_e on the global ratio R_g can be expected from Eq. (14) in the same way as from Eq. (12), i.e., the larger R_g, the smaller σ/R_e for fixed $\langle \pi \rangle$ and other parameters. It is shown that σ/R_e increases considerably with w_k. As argued in Ref. [21], the main contribution to the fluctuations of the K/π ratio comes from the fluctuation of multiplicity of kaons which are the particle species with fewer mean multiplicity. Normally, the relative multiplicity fluctuation for such particle species is larger. So in following calculations, three values of $w_K = 2.0, 2.5, 3.0$ are chosen to present the dependence of σ/R_e on the width of kaon multiplicity distribution. Because the mean ratio is independent of w_K, the calculated R are the same and close to 1.0 within 2.5% with the three choices of w_K, and are given also in Fig. 2. However, σ/R_e increases considerably with w_K, as shown in Fig. 3. In those analyses in [16, 17, 18] σ/R_e from the mixed events is about 23% with $\langle \pi \rangle$=270.13. Then from Fig. 3 one can see that the calculation from the MNB distribution can give value of σ/R consistent with the experimental result from the mixed events when w_k is suitably chosen (less than 2.5).

However, the value of w_K in real experiment is unknown yet and may be different from 2.5 significantly. Thus, the calculated result from pure statistical consideration cannot at present be compared directly with that from the mixed events out of experimental data. The possible difference between them may have deep physical implications. This may show that there exist some other physical effects in the event-by-event K/π distribution, which might be the correlation between the productions of kaons and pions. Such effects should be studied theoretically and experimentally.

As a summary, the statistical influence on the K/π ratio is studied in an event-by-event way. It is shown that the distribution of the ratio in the pure statistical case is Gaussian with mean and width depending on the mean multiplicities of kaons and pions. It is interesting to further study the effect on the ratio from correlations between the productions of kaons and pions and the influence of other dynamical contributions.

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[1] J.C. Collins and M.J. Perry, Phys. Rev. Lett. **34**, 151 (1975); E.V. Shuryak, Phy. Rev. **C61**, 71 (1980); ibid. **C115**, 151 (1984).
[2] S.A. Bass, M. Gyulassy, H. Stöker and W. Greiner, J. Phys. **G25**, R1 (1999).
[3] C.P. Singh, Phys. Rep. **236**, 147 (1993).
[4] Nucl. Phys. **A638**, Proceedings of the '97 Quark Matter conference, Tsukuba, Japan, 1-5 December, 1997, edited by T. Hatsuda, Y. Miake, S. Nagamiya and K. Yagi.
[5] P. Koch, B. Müller and J. Rafelski, Phys. Rep. **142**, 167 (1986).
[6] L.P. Cserni, *Introduction to Relativistic Heavy Ion Collisions*, J. Wiley, New York, 1994.
[7] M. Gazdzicki and St. Mrowczynski, Z. Phys. **C54**, 127 (1992).
[8] M. Gazdzicki, A. Leonidov and G. Roland, Eur. Phys. J. **C6**, 365 (1999).
[9] M.L. Cherry et al. (KLM Collab.), Acta Phys. Pol. **B29**, 2129 (1998).
[10] M. Bleicher, M. Belkacem, C. Ernst et al., Phys. Lett. **B435**, 9 (1998).
[11] St. Mrowczynski, Phys. Lett. **B439**, 6 (1998).
[12] St. Mrowczynski, Phys. Lett. **B459**, 679 (1999).
[13] F. Liu, A. Tai, M. Gazdzicki et al., Eur. Phys. J. **C8**, 649 (1999).
[14] S. Afanasiev et al. (NA49 Collab.), Nucl. Instr. Meth. **A430**, 210 (1999).

[15] R. Stock, Page 507 in Proceedings of NATO Advanced Study Workshop on Hot Hadronic Matter: Theory and Experiment, Divonne-les-Bains, France, 27 Jun - 1 Jul 1994.

[16] G. Roland, talk given in Hirschegg '97 Workshop on *QCD Phase Transitions*, Hirschegg, Germany, 13-18 Jan. 1997, NA49 Note Number 116.

[17] F. Sikler, talk given in *Quark Matter '99*, Torino, Italy, 10-15 May, 1999, NA49 Note Number 200.

[18] S.V. Afanasiev, (NA49 Collab.), NA49 Note Number 221, submitted to Phys. Rev. Lett.

[19] N. Suzuki, M. Biyajima, and G. Wilk, Phys. Lett. **B268**, 447 (1991); N. Suzuki, M. Biyajima and N. Nakajima, Phys. Rev. **D53**, 3582 (1996).

[20] P.V. Chliapnikov and O.G. Tchikilev, Phys. Lett. **B242**, 275 (1990); P.V. Chliapnikov and O.G. Tchikilev and V.A. Uvarov, Phys. Lett. **B352**, 461 (1995).

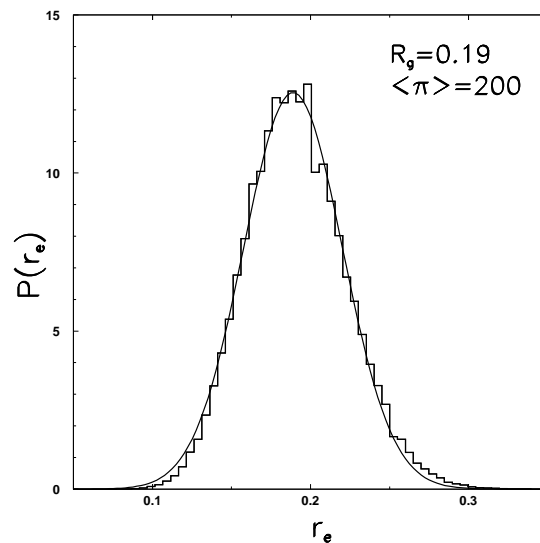[21] S. Jeon and V. Koch, Phys. Rev. Lett. **83**, 5435 (1999).

FIG. 1: The distribution of K/π ratio from Monte Carlo simulation for the independent production of kaons and pions with the global K/π ratio $R_g = 0.19$ and mean pion multiplicity $\langle \pi \rangle = 200$. Both the distributions for kaons and pions are assumed to be Poissonian. The solid curve is a Gaussian fit to the distribution.
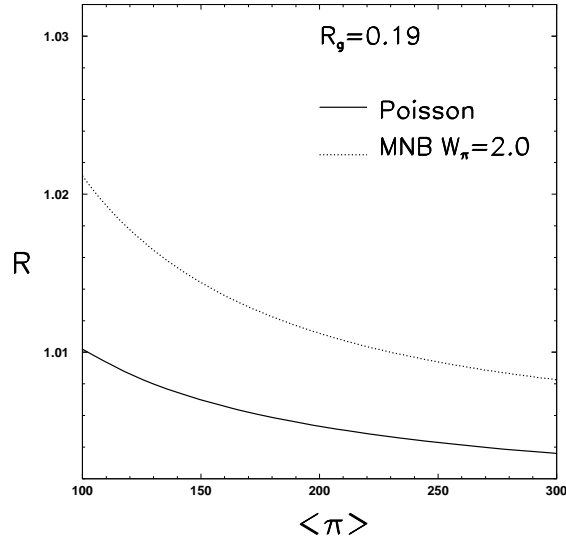
FIG. 2: The mean K/π ratio from an event-by-event analysis relative to R_g as a function of $\langle \pi \rangle$ for the cases with the Poisson and the MNB distributions for pion multiplicity.
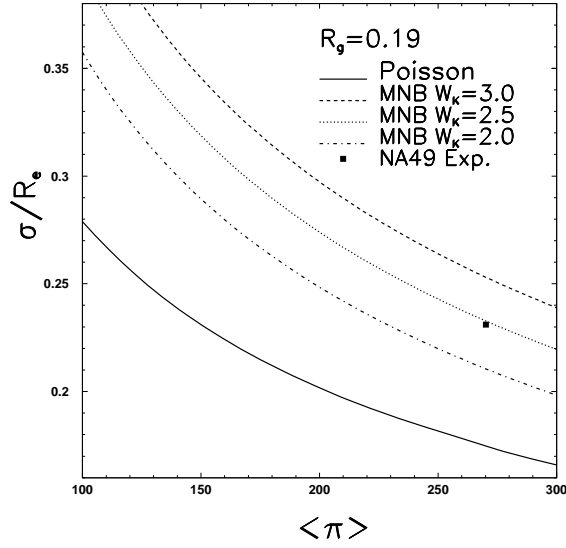

FIG. 3: The relative variance of the distribution of K/π ratio as a function of $\langle \pi \rangle$ for given $R_g = 0.19$ for the cases with the Poisson and the MNB distributions of the multiplicities of kaons and pions. The experimental point from [18] is given by a solid square.